

Notes 4.2 – Transformation of Rational Functions

Warmup

Find the zeroes (roots) for each polynomial.

1. $p(x) = (x + 4)(x - 2)(x - 7)$

$$x = -4, 2, 7$$

2. $p(x) = (2x - 6)(8x - 1)(x - 5)$

$$x = 3, \frac{1}{8}, 5$$

3. $p(x) = (9x + 3)(x^2 - 9)$

$$x = -\frac{1}{3}, 3, -3$$

4. $p(x) = x^2 + 25$

$$x = \pm 5i \quad (\text{no real roots})$$

Use the values that you found in #1-4 to help you write the domain of each function.

5. $q(x) = \frac{1}{(x+4)(x-2)(x-7)}$

$$D: x \neq -4, 2, 7$$

6. $q(x) = \frac{1}{(2x-6)(8x-1)(x-5)}$

$$D: x \neq 3, \frac{1}{8}, 5$$

7. $q(x) = \frac{1}{(9x+3)(x^2-9)}$

$$D: x \neq -\frac{1}{3}, 3, -3$$

8. $q(x) = \frac{1}{x^2+25}$

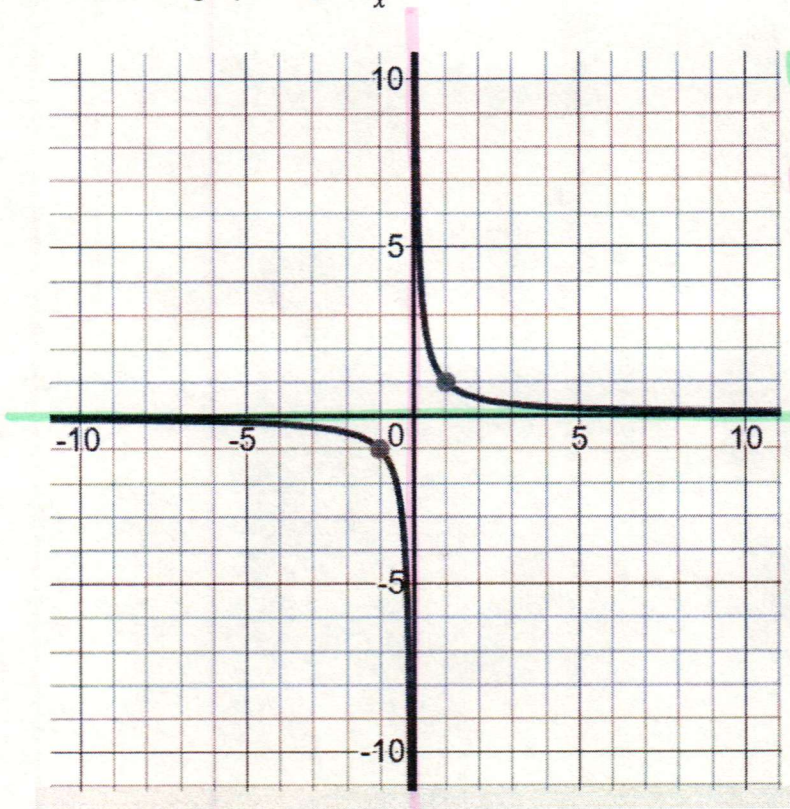
$$D: \mathbb{R}$$

9. What is general rule for finding the domain of a rational function?

The domain can be any number except a value that makes the denominator equal to zero.

Investigation – Transformations of Rational Functions

Here is the graph of $y = \frac{1}{x}$



Horizontal Asymptote: $y = 0$

Vertical Asymptote: $x = 0$

Anchor Points:

$(1, 1)$ and $(-1, -1)$

$(\frac{1}{2}, 2)$ and $(-\frac{1}{2}, -2)$

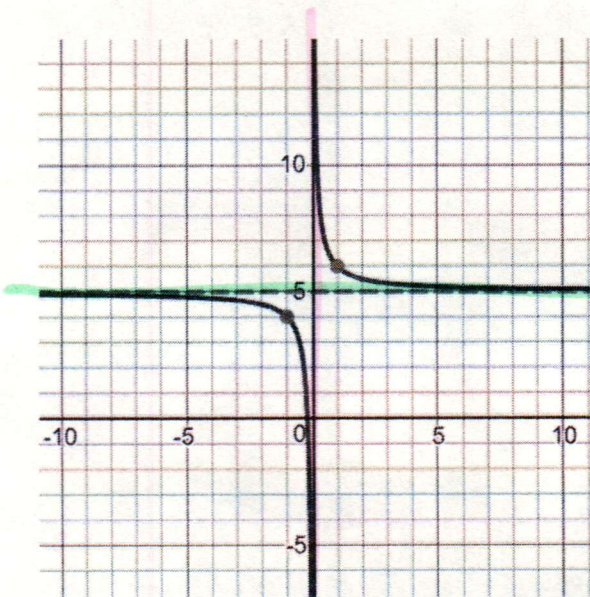
$(2, \frac{1}{2})$ and $(-2, -\frac{1}{2})$

Domain: $x \neq 0$

Range: $y \neq 0$

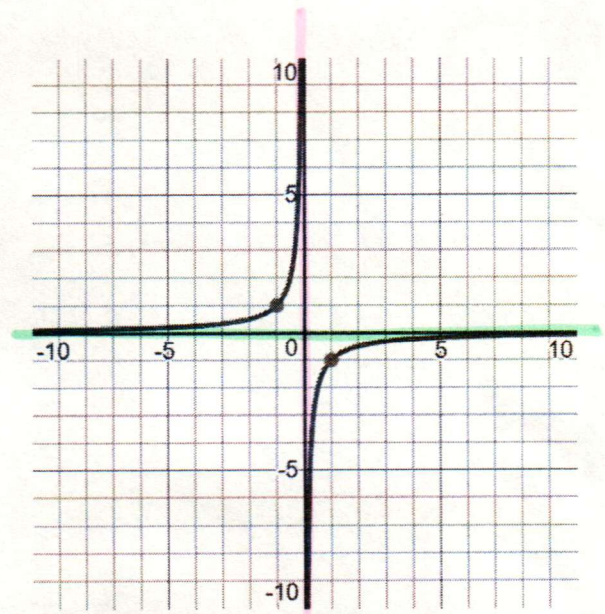
Now use what you know about transformations and the anchor points from above to write equations for each given graph or description.

a.



up 5
 $y = \frac{1}{x} + 5$

b.

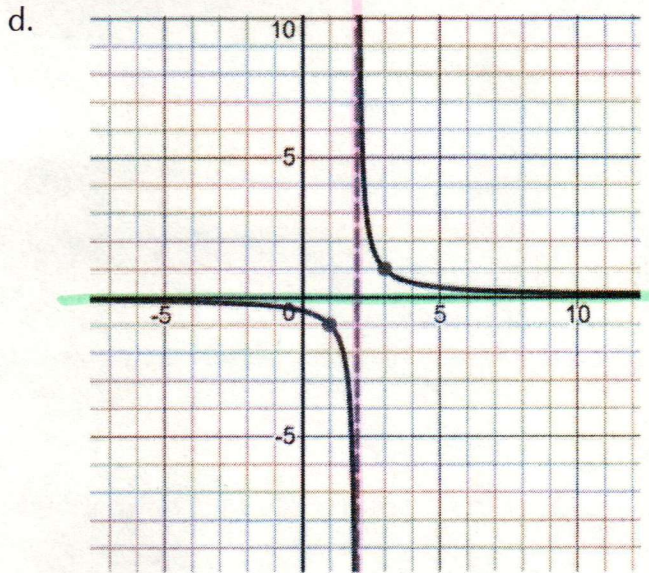


reflect
 $y = -\frac{1}{x}$

- c. Vertical Asymptote: $x = -3$
 Horizontal Asymptote: $y = 0$
 Points: $(-2, 1)$ and $(-4, -1)$
 y-int: $(0, \frac{1}{3})$

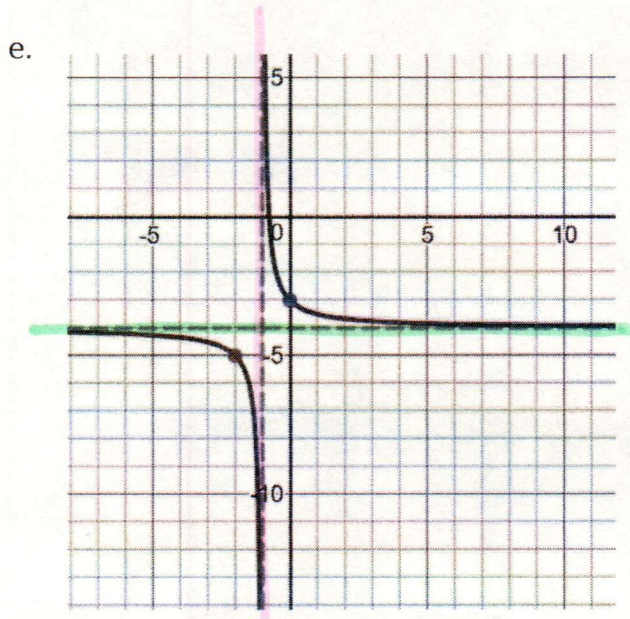
left 3

$$y = \frac{1}{x+3}$$



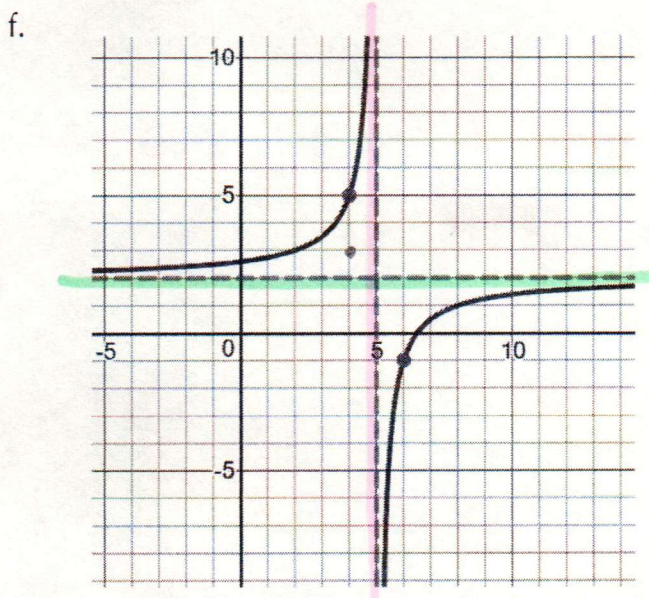
right 2

$$y = \frac{1}{x-2}$$



left 1
down 4

$$y = \frac{1}{x+1} - 4$$



stretch 3
right 5
up 2
reflect

$$y = \frac{-3}{x-5} + 2$$

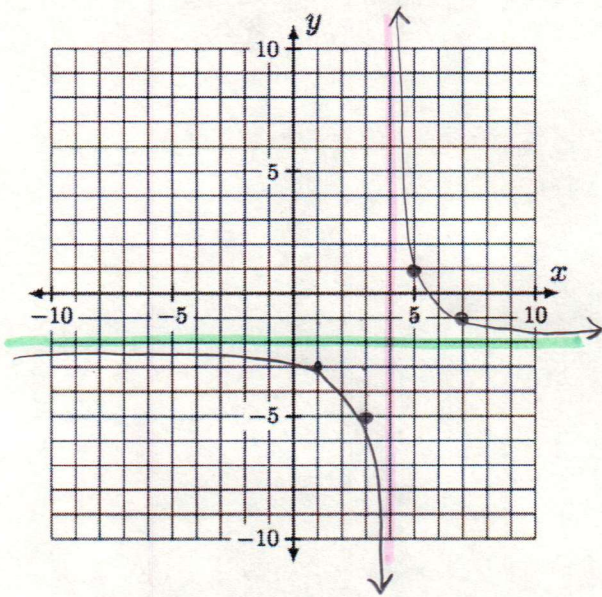
- g. Match the transformation with the equation.

$y = \frac{1}{x+b}$	<u> C </u>	A) Reflection over the x -axis.
$y = b + \frac{1}{x}$	<u> B </u>	B) Vertical shift of b , making the horizontal asymptote $y = b$.
$y = \frac{b}{x}$	<u> D </u>	C) Horizontal shift left b , making the vertical asymptote $x = -b$.
$y = \frac{-1}{x}$	<u> A </u>	D) Vertical stretch by a factor of b
$y = \frac{1}{x-b}$	<u> E </u>	E) Horizontal shift right b , making the vertical asymptote $x = b$.

Without using technology, graph the given functions using what you know about transformations.

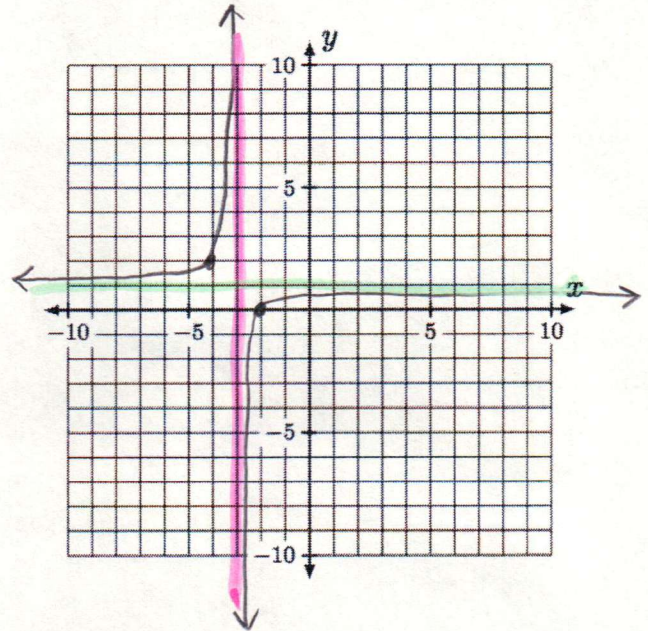
h. $y = -2 + \frac{3}{x-4}$

down 2
right 4
stretch 3



i. $y = 1 - \frac{1}{x+3}$

up 1
reflect
left 3



Describe the features of the function:

$$y = k + \frac{b}{x-h}$$

Vertical Asymptote: $x = h$

Horizontal Asymptote: $y = k$

Vertical Stretch: b

Domain: $x \neq h$

Range: $y \neq k$ or $(-\infty, k) \cup (k, \infty)$

or

$(-\infty, h) \cup (h, \infty)$

Anchor Points: $(h+1, k+b)$

and

$(h-1, k-b)$